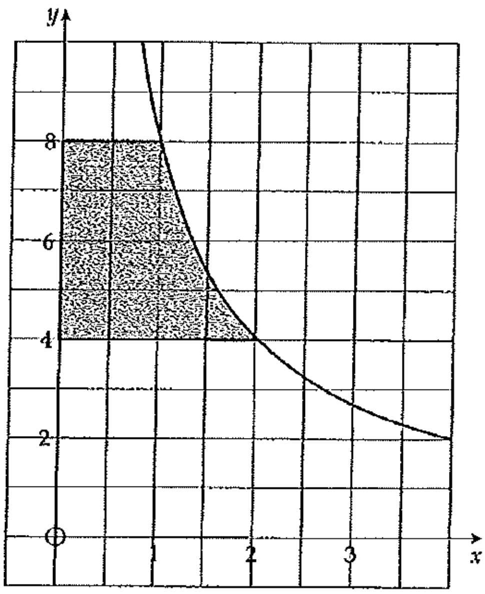
Volumes of Revolution – rotation about the y-axis

When finding the volume of the solid formed when rotating a function about the y-axis, you need to rearrange the function to make x the subject, and then modify the volume formula to find the integral of x^2 with respect to y.

Rule: $V = \pi \int_a^b x^2 \; dy$

Example 1: Find the volume of the solid formed by rotating the shaded portion of the function $y = \frac{8}{x}$ around the y-axis.



Rearrange: $y = \frac{8}{x} \quad \rightarrow \quad x = \frac{8}{y} \quad \rightarrow \quad x^2 = \frac{64}{y^2}$

$$V = \pi \int_a^b x^2 \; dy$$

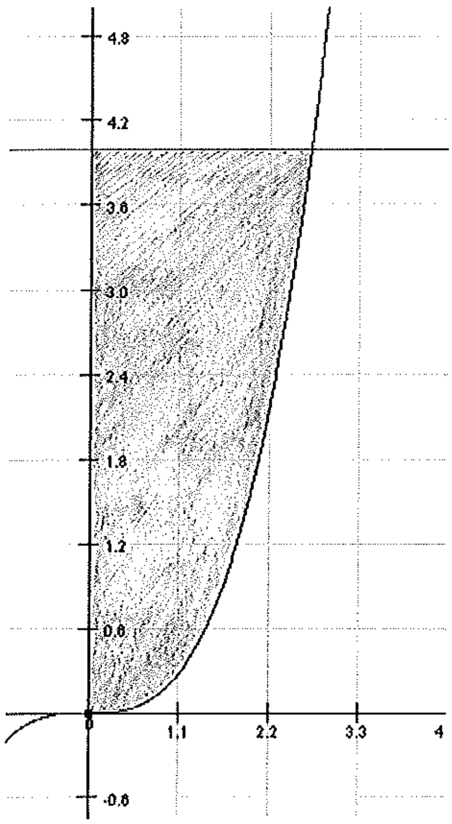
$$= \pi \int_4^8 \frac{64}{y^2} \; dy$$

$$= \pi \int_4^8 64y^{-2} \; dy$$

$$= \left[64\frac{y^{-1}}{-1}\right]_4^8$$

$$= 8\pi \text{ units}^3$$

Example 2: Find the volume of revolution when the function $y = 0.2x^3$ is rotated about the y-axis between $y = 0$ and $y = 4$.



Rearrange: $y = 0.2x^3 \quad \rightarrow \quad x^3 = \frac{y}{0.2} = 5y \quad \rightarrow \quad x = (5y)^{1/3} \quad \rightarrow \quad x^2 = (5y)^{2/3}$

$$V = \pi \int_a^b x^2 \; dy$$

$$= \pi \int_0^4 5^{2/3} y^{2/3} \; dy$$

$$= 5^{2/3}\pi \int_0^4 y^{2/3} \; dy$$

$$= 5^{2/3}\pi \left[\frac{y^{5/3}}{5/3} \right]_0^4$$

$= 55.554$ units^3

Delta Ex 21.2 Q1, 2a, 2b, 5, 6, and 7.